

**CAN TWO PERSON ZERO SUM GAME THEORY  
IMPROVE MILITARY DECISION-MAKING COURSE OF  
ACTION SELECTION?**

**A Monograph  
by  
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## ABSTRACT

CAN TWO PERSON ZERO SUM GAME THEORY IMPROVE MILITARY  
DECISION-MAKINGCOURSE OF ACTION SELECTION? By LIEUTENANT COLONEL  
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Colonel (retired) Oliver G. Haywood suggested in his brilliant 1954 article, "*Military Decisions and Game Theory*" that game theory techniques were relevant to preparing the military commander's estimate of the situation. He based his article on work he had done as a student at the Air War College in 1950. Colonel Haywood demonstrated the utility of game theory by analyzing two World War II military operations. In each case, he examined the various friendly courses of action and compared them with enemy courses of action to determine the value of the predicted outcome. He concluded that military decision-making doctrine was similar to solving two-person zero-sum games. Finding the optimal solution for a two person zero sum game is not the challenge. With an understanding of how to solve a two person zero sum game matrix, the challenge remains to apply these concepts to the military decision-making process. How does a staff take a complex military situation and reduce it to a two-person payoff matrix? This study proposes a ten-step method to determine the military worth values of a two person zero sum game matrix. The proposed ten-step method organizes the information obtained in mission analysis and allows the commander to determine the optimal strategy for a military situation. These steps ought to be added to the current army decision-making process outlined in Army Field Manual: *FM 101-5, Staff Organization and Operations*, course of action comparison. This study provides a summary of the steps for determining the optimal solution for a two person zero sum game. The summary can be used as a memory aid in solving two person zero sum games. The appendixes provide the details of the proposed method for determining military worth for a two person zero sum game.

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## INTRODUCTION

Colonel (retired) Oliver G. Haywood suggested in his brilliant 1954 article, “*Military Decisions and Game Theory*” that game theory techniques were relevant to preparing the military commander's estimate of the situation. He based his article on work he had done as a student at the Air War College in 1950. Colonel Haywood demonstrated the utility of game theory by analyzing two World War II military operations. In each case, he examined the various friendly courses of action and compared them with enemy courses of action to determine the value of the predicted outcome. He concluded that military decision-making doctrine was similar to solving two-person zero-sum games. Colonel Haywood's assertion encouraged the operations research community to develop quantitative methods to enhance decision-making.

Business and government bureaucracies have gone to great lengths to use operations research methods to improve their productivity and decision-making. Advances in technology have integrated automation and simulations to the point that nearly every corporate decision relies on automation at some level. There are obvious differences between corporate goals and military objectives. However, both deal with decision-making under uncertainty. Military and corporate decision-making involves complex and ambiguous problems. Both manage finite resources and complex interrelated systems. Seldom can the decision maker obtain perfect understanding of the situation and the effects of a decision. Business has embraced quantitative methods and game theory to improve their analysis and decision-making. Based on the benefits of game theory to the business world, military decision-making may also benefit by applying game theory.

The military decision-making process is a tool to assist the commander and staff in developing estimates and plans.<sup>1</sup> In 1950, military decision-making centered on the estimate of the situation. This was a five-step process that included: 1) The mission, 2) The situation, and

courses of action, 3) Analysis of the opposing courses of action, 4) Comparison of available courses of action, and 5) The Decision.<sup>2</sup> Current Army Doctrine portrays the military decision-making process (MDMP) model as a seven-step process.<sup>3</sup> The two steps added since 1950 are: first, receive the mission, and last, orders production. Neither of these steps affects the process of decision-making utilized in 1950. Minor changes have occurred in the titles of steps two and five, but the process remains intact.<sup>4</sup> It seems amazing that more than fifty years have past and the decision-making model steps remain relatively unchanged

Weapons systems now have capabilities that were only dreams of inventors in 1950, yet no microchip exists to determine the optimal solution of a military conflict for a military commander. Despite many changes in military equipment, doctrine, training, and organization, the military decision-making process has remained relatively unchanged. There is still no doctrinally defined method for selecting a course of action. The question, now, is whether COL Haywood's suggestion can improve military decision-making; that is, does applying the two-person zero sum game theory to course of action selection improve the decision process?

There are still obviously many differences between military plans and operations and corporate decisions. It is difficult to imagine a complex military operation as a simulation or an equation with a clear numerical solution. Colonel Haywood recognized this fact and suggested that the military worth of an encounter could be determined in a manner similar to the corporate

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<sup>1</sup> United States Army, Field Manual 101-5 (Washington, D.C.: Government Printing Office, 1997), 5-1.

<sup>2</sup> O.G. Haywood, "Military Decisions and Game Theory," *Journal of the Operations Research Society of America* 4 (November 1954), 367.

<sup>3</sup> United States Army, Field Manual 101-5 (Washington, D.C.: Government Printing Office, 1997), 5-3. The seven steps are: 1) Receipt of the mission, 2) Mission analysis 3) Course of action development, 4) Course of action analysis, 5) Course of action comparison, 6) Course of action approval, and 7) Orders production.

<sup>4</sup> Ibid., The 1950-second step titled the situation and courses of action changed to mission analysis and course of action development. The 1950 step five, the decision, is now course of action approval and orders production.

bottom line. Further, he argued that game theory could improve doctrine by providing a method for selecting the military course of action with the greatest probability of success. However, no doctrinal approach exists to determine the military worth of an encounter, in order to apply game theory.

To bring game theory to military course of action selection, this study proposes a ten-step method for determining the values of military worth for a two person zero sum game. These steps augment the current army decision-making process outlined in Army Field Manual *FM101-5, Staff Organization and Operations*, course of action comparison, page 5-24. They serve to answer the challenge, correctly identified by Colonel Haywood over fifty years ago, to develop a process to determine the military worth of the effects produced by a military confrontation in order to apply two person zero sum game theory.<sup>5</sup> The proposed ten-step method provides a model, which the commander can use to organize the information obtained in mission analysis to simplify the problem. This model enables the commander to apply game theory to improve his military decision-making. This model supports the mission analysis process and aids course of action development, comparison, and selection. It utilizes the two person zero sum game to focus on the effects generated by opposing courses of action. It is a capabilities-based analysis dependant upon the military decision maker's understanding of the scenario. This ten-step process may improve military decision-making and encourage other imaginative solutions to complex problems.

Analysis of the military decision-making process and assessments of the course of action analysis selection process suggest that the concepts of two person zero sum game theory can improve military decision-making. This conclusion was developed by: 1) examining the concepts of game theory; 2) conducting a historical analysis of the Tannenberg Campaign of 1914 utilizing

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<sup>5</sup> O.G. Haywood, "Military Decisions and Game Theory", *Journal of the Operations Research Society of America* 4 (November 1954), 380.

game theory; and 3) by analyzing the value of game theory to the military decision-making process. Before looking at a historical example and examining the value of game theory to military decision-making, it is important to understand the concepts associated with a two person zero sum game.

## Solving Two Person Zero Sum Games.

A decision situation can be represented as a two person zero sum game if any gain obtained by one party results in a similar loss by his competitor. In terms of game theory, this is the zero sum assumption. This means the gain for one player comes at a loss to the other player. A way to visualize this is to consider the pot in a poker game. The winner takes all, at the expense of the other player. Game theory further assumes the two players are rational actors and each is trying to maximize his gain or minimize his loss. These assumptions closely resemble the assumptions of the military decision-making process. Prior to developing the military applications of game theory, it is important for the sake of clarity to understand game theory. Player One, the friendly player or Blue Player, is the row player. Player Two, the enemy player or Red Player, is the column player. All the values in the matrix are Blue Player payoffs. Any negative values indicate a Blue Player loss and a gain for the Red Player. Any positive values indicate a Blue Player Gain and a Red Player loss. Consider a game with two options available for each player. The situation creates a 2 x 2 matrix as shown below in figure 1. The arbitrarily selected values in the matrix reflect the payoffs associated with the two courses of action.

		Red Player, Player 2, Column Player	
		COA 1	COA 2
Player, , Player	COA A	2	4
	COA B	2	0

Figure 1, example 1

In this example, figure 1 above, when the Blue Player selects course of action A and the Red Player selects course of action one, the Blue Player receives a payoff of two. Similarly, when the Blue Player selects course of action A and the Red Player selects course of action two, the Blue Player receives a payoff of four. If the Blue Player selects course of action B and the Red Player selects course of action one, he receives a payoff of two. Similarly, if the Blue Player selects course of action B and the Red

player selects course of action 2 the Blue player receives a payoff of zero. The payoff values for the Red Player are equal to the Blue Player outcomes multiplied by negative one. Thus, all payoffs are a gain for one player and a loss for the other player.

The first step in determining the optimal strategy for each player is to check each course of action for dominance over the other courses of action. The theory of dominance states that if a course of action is better than another course of action for all combinations; eliminate the lesser course of action from further consideration. Simply stated, if a player could do better with course of action one than two, he would never pick course of action two and could eliminate it from further consideration. This reflects the second assumption of game theory that all actors are rational actors and working to maximize their profits or minimize their losses. Dominance becomes important when dealing with complex problems because it allows the player to simplify the problem by eliminating some courses of action from consideration. Applied to military decision-making, any tool that assists in simplifying a decision is worth exploring. From the Blue Player's standpoint in this example, he would always do as well or better by selecting course of action A over course of action B, regardless of the other player's actions. The Blue Player should never select course of action B. Therefore, eliminate course of action B from further consideration. Figure 2 below contains the reduced matrix.

		Red Player, Player 2	
yer,		COA 1	COA 2
	COA A	2	4
	COA B	2	0

Figure 2, example 1

Similarly, the Red Player can use the theory of dominance to simplify his choices. Remembering that the values in the table are the payoffs in terms of the Blue Player, the Red Player would be better off losing two than losing four. Therefore, the Red Player would be better off selecting course of action one than he would be selecting course of action two regardless of the Blue player's choice. The matrix further reduces to one course of action for each player as displayed in figure three below.

		Red Player, Player 2	
yer,		COA 1*	COA 2
	COA A*	2	4
	COA B	2	0

Figure 3, example 1

The optimal strategy (represented by a \* in figure 3 above) for the Blue Player would be course of action A, and for the Red Player course of action one under all conditions. Deviation from this strategy over time results in worse performance than could be achieved by executing the optimal strategy for either player. The fact that there is one best course of action for each player means this optimal strategy is a pure strategy for each player. The assumption of a zero sum game does not guarantee that the game is fair. The Blue Player wins every turn of the game. The Red Player's loss equals the inverse of the Blue Player's gain. Therefore, the game is a zero sum game. The game value, however, is two. Since this is a positive value, the game favors the Blue Player. This example illustrates the value of dominance in simplifying course of action selection. Dominance alone rarely reduces a matrix to a single course of

action for each player in complex situations. The example in figure 4 below explores a similar two person zero sum game with two options available for each player. Unlike the first example, dominance will not reduce the matrix to a single course of action.

Blue Player, Player 1	Red Player, Player 2	
	COA 1	COA 2
	COA A	COA B
	4	-3
	-2	5

Figure 4, example 2

First, apply the concept of dominance to eliminate courses of action and simplify the problem. Remembering that the values in the payoff matrix above are in terms of payoffs to the Blue Player, negative values are gains to the Red Player. If the Red Player selected course of action one, the Blue Player would obtain the best result by selecting course of action A. However, if the Red Player selected course of action two, then the Blue Player would obtain the best result by selecting course of action B. Therefore, dominance does not reduce the matrix for the Blue Player. Similarly, if the Blue Player selected course of action A, the Red Player would be better off gaining three than losing four and would select course of action two. If the Blue Player selected course of action B, the Red Player would do best by selecting course of action one. In this example, dominance did not reduce any of the available courses of action.

When dominance does not simplify the decision, then another concept may help. The concept of Maximin and Minimax helps to determine the optimal solution for each of the players.<sup>1</sup> The Maximin is the maximum of the minimum values. The minimax is the minimum of the maximum values. The Maximin concept is also known as a pessimistic or conservative approach because it determines the

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<sup>1</sup> Richard E. Trueman, "*Quantitative methods for Decision-Making in Business* (Illinois: The Dreyden Press, 1981), 169.

optimal strategy assuming the worst-case scenario. This technique solves for an optimal strategy for each player and searches for a saddle point. A saddle point simply occurs when the two players' optimal strategies intersect in the matrix in the same box. At this point, neither player can do any better than they would if they played their optimal strategy. In fact, if one player deviates from their optimal strategy, he will do worse in the end. The term saddle is used to illustrate the point that if either player departs from the optimal strategy for sitting in a saddle, the center position, then deviation from this position left or right results in falling off the side of the horse. The concept suggests that your opponent will always seek to maximize his gains while minimizing his opponent's gains. This concept is referred to as a pure strategy because each player only plays one course of action rather than switching between several courses of action. This Maximin method, or Wald's Criterion, provides the Blue Player with the optimal solution assuming his opponent plays well.<sup>2</sup> The Blue Player seeks to take the maximum of the minimum possible (Maximin) returns. Considering the fact that the Red Player is trying to minimize the Blue Player's gains, it follows that the Blue Player should select the minimum of the maximum (Minimax) gain for each course of action available to him.

Analysis of figure four above for the Blue Player shows that the minimum payoff for course of action A is a loss of three. The minimum payoff for course of action B is a loss of two. Comparing the two minimums and selecting the maximum of the minimums (Maximin) results in selecting course of action two as the Maximin solution for the Blue Player because a loss of two is better than a loss of three. These results are below in figure five as the Maximin. In other words, if the Blue Player assumed a pessimistic outcome the worst he could do is to lose three by selecting course of action A or lose two by selecting course of action B. If he adopted a conservative or pessimistic strategy of picking the best of the worst possible outcomes he would select course of action B, knowing that he could do no worse than lose two. Similarly, the Blue Player wants to maximize the Red Player's losses and minimize the Red Player's gains. A loss of two minimizes the best possible gain for the Red Player.

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<sup>2</sup> Ibid., 169.

The Red Player will select the maximum values for each column corresponding to each Blue course of action. In this case four is greater than a loss of two and, therefore, four is the maximum for course of action one. This takes into account that the Blue player would prefer to maximize his gains while minimizing his losses. For course of action two, the best outcome results from course of action two. Taking the minimum of the maximum (Minimax) values, course of action one is the optimal solution for the Red Player. In other words, the Red player will expect the Blue Player to be a rational player and attempt to maximize his gains. The Blue Player therefore, would select the course of action corresponding to the maximum payoff. Therefore, the Red Player achieves the Minimax solution when he selects the course of action that returns the minimum of the maximum values to minimize the gains for the Blue Player. These results are below in figure five as the Minimax.

		Red Player, Player 2		
		COA 1	COA 2	Minimum Row payoff
Blue Player, Player 1	COA A	4	-3	-3
	COA B	-2	5	-2
	Maximum Column payoff	4	5	
		Minimax	Maximin	

Figure 5, example 2

In figure five above, the Maximin solution for the Blue Player is to play course of action B and the Minimax solution for the Red Player is to play course of action one. The two strategies do not intersect in the same box of the matrix so there is no saddle point or pure solution. Because there is no saddle point, there is no pure strategy for this game. There is a mixed solution for each player that will provide the optimal solution for this game. Reviewing the matrix in figure five, one can see that the Blue Player could benefit from knowing which course of action the Red Player intends to play. If the Blue Player knew, the Red Player selected course of action one, he would pick course of action A and gain four rather than lose two. Similarly, the Blue Player benefits from keeping his intentions from the Red Player. The Blue Player also benefits from deceiving the Red Player to his intentions. These concepts are similar to the military concepts of espionage, secrecy, and deception. Both Players gain an advantage if they can learn the others intentions. Since there is no saddle point this is a closed game because the players gain by keeping their intentions from the other. The optimal mixed strategy for each player can be determined graphically since only two courses of action exist for each player. The graphical method is limited to two courses of action because there are only two dimensions on a graph. The graphical solution below demonstrates the concepts for solving two person mixed strategies.

Starting with the Blue Player, his or her options are to select courses of action A or B. One can represent these options graphically by plotting the payoff values on opposite ends of a line. The line constructed by connecting these points represents the combination between the two strategies that compare the opponent's actions. From the values in figure five, one can make the following analysis. If the Blue Player employs a pure strategy of course of action A, he would expect a value of four, if the Red Player chose course of action one, or a value of negative three if the Red Player selects course of action two. Similarly, if the Blue Player selects course of action B he expects a payoff of negative two, if the Red Player selects course of action one, and five if the Red Player selects course of action two. Figure six below illustrates this discussion.

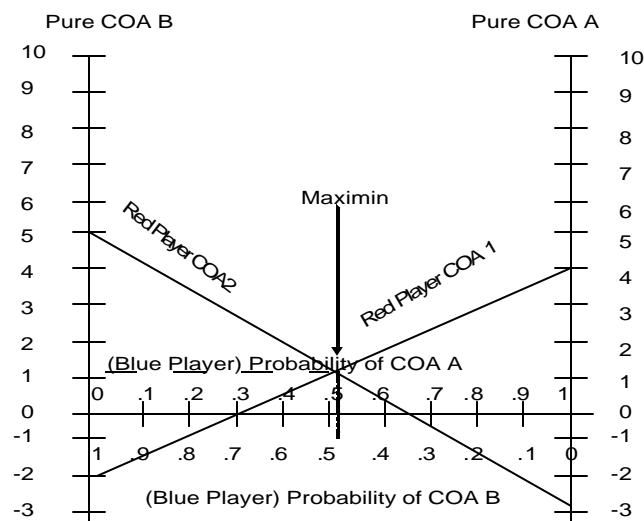


Figure 6, Blue Player optimal solution

The point where the Red Player's courses of action cross represents the optimal solution for the Blue Player. This is the Maximin point or maximum payoff of the minimum values for the Blue Player. From figure six above, one can see that this value is greater than one and is greater than the value identified in figure five previously as the maximin of negative two. This is the value of the game. The value of the game can also be found algebraically by substituting the probabilities from the optimal

solution into the equation of the lines that intersect to form the Maximin point. The dotted vertical line originating at the Maximin point indicates the probabilities associated with the optimal solution. To optimize his or her gain, the Blue Player should select course of action A approximately 52 percent of the time and course of action B approximately 48 percent of the time.

The optimal solution for the Red Player can be determined graphically using the payoff values from figure five. The Red Player also has two courses of action to consider. The expected payoff (loss) if the Red Player employs course of action one is four, if the Blue Player employs course of action A, or a gain of two if the Blue Player employs course of action B. Similarly, the payoff values for the Red Player are determined for employing a pure strategy of course of action two. These values are a gain of three, if the Blue Player employs course of action A, or a loss of five if the Blue Player employs course of action B. The lines that connected these points represent the payoff resulting from a combination of mixing courses of action one and two.

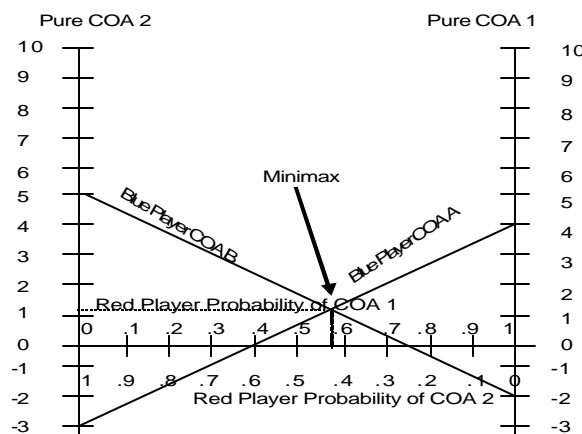


Figure 7, the Red Player graphical solution

The point where the two lines intersect represents the optimal solution for the Red Player and the Minimax. The horizontal line that intersects with the Minimax and appears to approach one again represents the value of the game. The vertical line that intersects the Minimax point indicates the

approximate probabilities associated with the Red Player's optimal solution. The graphical solution indicates that the Red Player should play a mixed strategy of course of action one approximately 56 percent of the time and course of action two 44 percent of the time. Figure seven above graphically portrays the solution. With this basic understanding of the concepts required to solve a two person zero sum game, it is possible to explore the application of game theory to military decision-making.

## Game Theory Applied to Tannenberg Example

Understanding how to solve a two person zero sum game matrix is not enough to establish the relevance of game theory to military decision-making. The challenge remains to apply these concepts to the military-decision-making process. How does one take a complex military situation and reduce it to a two-person payoff matrix? This chapter proposes a ten-step method to determine the values of military worth in a two person zero sum game matrix. The term military worth represents the payoff values from a military situation applied to a two person zero sum game matrix. In the previous examples, the values were assigned arbitrarily. For game theory to be applicable to the military decision-making process, the commander must translate careful mission analysis into military worth values in a payoff matrix. Colonel Haywood's paper illustrates the value of two person zero sum game theory for military decision-making with historical examples. A contemporary military problem, a Marine Corps Gazette's scenario, *Battle at Dadmamian Swamp*, serves the same purpose.<sup>1</sup> The United States Marine Corps Tactical Decision Games (TDG), developed by CPT John Schmitt provides numerous scenarios for applying game theory. Several Marine Corps Officers are familiar with tactical decision games from their readings of the monthly periodical, the *Marine Corps Gazette*. These scenarios also have the advantage of being short and easy to understand. Other military gaming scenarios exist that can be adapted to examine game theory. However, complete treatment of all possible game scenarios and techniques is beyond the scope of this study. CPT Schmitt describes a notional battle in The *Battle at Dadmamian Swamp* scenario in the *Marine Corps Gazette* based loosely on the German situation at the Tannenberg Campaign in August

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<sup>1</sup> John Schmitt, *Mastering Tactics: a Tactical Decision Game Workbook* (Quantico: Marine Corps Association, 1994), 36.

1914.<sup>2</sup> This chapter will utilize the proposed ten-step method to analyze the Battle of Tannenberg to illustrate the value of game theory to the military planner

A brief historical synopsis of the Tannenberg scenario reveals the German forces faced the Russian forces on the Russian Western front. German forces were numerically inferior to the Russian forces. The Russian forces were organized into two armies separated by the Mansurian Lakes region. Rennenkampf commanded the First Russian Army in the North and Samsonov commanded the Second Russian Army in the South.<sup>3</sup> Together the Russian forces may have been able to overwhelm the German forces in a coordinated attack.

To analyze this scenario in terms of game theory, consider the German Commander Player One and the Russian Commander as Player Two. The German Commander's courses of action, Player One, reduce to the following four options: attack in the North and fix in the South; attack in the South and fix in the North; defend in place; and fall back West of the Vistula River and defend. Similarly, the Russian Commander, Player Two, has six courses of action: Rennenkampf can attack in the North, unassisted by Samsonov ; Samsonov can attack in the South, unaided by Rennenkampf in the North; They can conduct simultaneous separate attacks in the North and South; or they can attack in the North, fix in the South; or they can attack in the South and fix in the North; Lastly, they can defend and trade space for time against the German advance while preparing for a counterattack after the German army culminates.<sup>4</sup>

Given the defined courses of action, it is possible to represent each player's choices in a matrix. With a matrix established it is possible to utilize game theory to determine the optimal course of action for each player. It is important to note that the courses of action described above are based on the opposing player's capabilities, rather than on their intentions. Like the situation in the preceding chapter,

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<sup>2</sup> John Schmitt, *Mastering Tactics: a Tactical Decision Game Workbook* (Quantico: Marine Corps Association, 1994), 82.

<sup>3</sup> Ibid.

<sup>4</sup> Ibid., 83.

the Blue Player would do better by knowing the Red Players course of action. However, if the Blue player is deceived or incorrectly determines the enemy course of action the results would be worse. The pessimistic approach or Minimax theory assumes that your opponent will do their best to defeat you. Hence, it is better to consider what the enemy can do to you than what you think he will do to you. The pessimistic approach resembles the current military doctrine driving the intelligence preparation of the battlefield; the intelligence officer considers enemy capabilities as well as probable courses of action. The staff identifies the enemy capabilities before determining the likely enemy course of action<sup>5</sup>.

Although the German and Russian alternative courses of action are known, it is not yet possible to build the payoff matrix. First values must be assigned to represent the outcomes of the selected courses of action. These values must be determined in a systematic way for each course of action opposing each enemy course of action. These values can be determined in a number of ways. A conventional solution might compare the opposing force ratios suggested in the current army doctrine under course of action development.<sup>6</sup> Analyzing the force ratios and determining where the greatest advantages and weaknesses lie, aids the commander in developing courses of action. This is a necessary portion of the decision-making process but does not provide sufficient guidance to assign a numerical value to each combination of courses of action. Comparison of courses of action is the foundation of the commander's decision briefing. As stated in FM 101-5, too often "subjective conclusions are presented as objective results of quantifiable analysis".<sup>7</sup> The value of game theory to the military decision maker is that it focuses on the effects produced by the clash of opposing capabilities rather than on the courses of action. In the Tannenberg example, it is difficult to maintain a subjective bias toward a particular course of action without it becoming obvious. This is because the proposed ten-step method for determining military worth focuses on the effects of the actions rather than on the courses of action.

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<sup>5</sup> United States Army, Field Manual Number 101-5 (Washington, D.C., Government Printing Office, 1997), 5-7.

<sup>6</sup> Ibid., 5-24.

<sup>7</sup> Ibid.

Rather than using force ratios, the proposed ten-step method organizes the information obtained in mission analysis and allows the commander to determine the optimal strategy for a military situation. These steps conceivably can be added to the current army decision-making process outlined in Army Field Manual: *FM 101-5, Staff Organization and Operations*, course of action comparison, page 5-24. This is a recommended addition to the military decision-making process. The analysis conducted in the first four steps of the military decision-making process model provides the information to develop the military worth values for a payoff matrix. Again arbitrarily assigned values provide no improvement to the decision-making process. The analysis provides a common basis to evaluate all courses of action. The ten-step method provides the commander a mechanism to organize this information and simplify his decision-making. The Tannenberg example will illustrate the ten-step method and apply the information gained in mission analysis to determine each commander's best course of action. Appendix one of this study provides a summary of the steps described in chapter two to determine the optimal solution for a two person zero sum game. Use this summary as a memory aid in solving two person zero sum games. Appendix two of this study outlines the proposed ten-step procedure to develop the payoff matrix for the Tannenberg example. The reader is encouraged to review appendix two for greater appreciation of the proposed method of determining military worth for a two person zero sum game.

The list below presents the proposed steps to determine the values of military worth for a two person zero sum game payoff matrix. 1) Select the best-case friendly course of action to achieve a decisive victory. 2) Rank order the friendly courses of action from the best effects possible, to the worst effects possible. 3) Rank order the effects of enemy courses of action from the best to the worst across each row, in terms of the friendly player. 4) Determine whether the effects of the enemy course of action will result in a potential loss, tie, or win for the friendly player for every combination across each row. Write the result of this determination in each box. 5) Place the product of the number of rows multiplied by the number of columns in the box corresponding to the best-case scenario for Player One. 6) Rank order all of the combinations from best case to worst case that are marked win, from step 4. Use the

product obtained in step five as the highest number and assign values in descending order based on the best to the worst case effects anticipated for each combination marked win. 7) Place the number one in the box corresponding to the worst-case effects anticipated in terms of Player One. 8) Rank order all combinations marked loss, from step 4, from the worst case to the best case. 9) Rank order all remaining even combinations from the best case to the worst case. Finally, 10) Transcribe the matrix into a conventional format with each course of action listed in ascending order while maintaining the appropriate values for each combination. This will result in a 4 x 6 matrix representative of the commander's or military planner's synthesis of the effects anticipated for each course of action based on mission analysis.

First, consider the German courses of action. In this example, the German course of action with the greatest potential to achieve decisive victory is course of action one. Mission analysis indicates, amongst other things, that attacking in the North and defeating the First Army before it can unite with the Second Army in the South, will facilitate the piecemeal destruction of a numerically superior opponent. This analysis rests on the strength of the German interior lines and the capabilities of the German forces. The next best course of action relies on a strategy of attrition. A withdrawal to the Vistula River seeks to consolidate the front by defending along the natural obstacle of the Vistula River and to take full advantage of interior lines to defeat the enemy. This is a defensive strategy and allows more resources to be diverted to the German war with France. This course of action can still result in a German victory but is more conservative than course of action one. Again, it takes into consideration the German capabilities and seeks to maximize the advantage of secure lines of operations west of the Vistula River. Course of action two seeks to attack in the South, fix in the North and defeat the Russians piecemeal, as in course of action one. The primary difference between this course of action and the best course of action is the need to greatly reorganize the forces and move to the South, from the North. Additionally, the Russians in the North were victorious in a meeting engagement at Gumbinnen and this would require removing troops in front of an advancing enemy in this area. Historians also doubt the Germans were capable of holding in

the North against a Russian advance.<sup>8</sup> The effect of extended distances negated the advantages gained by interior lines for the Germans. It is interesting to note, that some historians claim course of action two was successful for the Germans because the Russians failed to press their advantage in the North.<sup>9</sup> Had the Russians advanced in the North, the Germans would not have been able to conduct their attack in the South.<sup>10</sup> Course of action three relies on attrition warfare for success but defends across the current frontier. Based on capabilities, the Germans cannot be strong everywhere and prevent the Russians from achieving mass and penetrating their lines. The distances involved, with the available force structure, negates the potential benefits of interior lines.

Having considered the German courses of action, the next step is to determine the best effects for each of the corresponding Russian courses of action. Without going into an extended discussion of the twenty-four combinations, the commander has determined that the best opportunity for a German success occurs when the Germans attack in the North, defeat the Russian First Army, then turns South, and defeats the Russian Second Army. Again, this is based on a capabilities estimate gained through the commander's mission analysis. Ranking the other courses of action from best-case to worst-case, considering the effects from the German perspective, the worst-case scenario occurs when the Germans attack in the South and the Russians employ a strategy of defense in depth and create the conditions for German defeat through culmination. Remembering that all payoffs in the matrix are in terms of benefit to the Germans, the worst combination has been assigned a value of one and the best possible combination given a value of twenty-four. Clearly, much mission analysis must be done before a matrix can be constructed that considers the enemy and friendly capabilities as well as the expected effects of the combination of these courses of action. Matrix construction cannot occur in a vacuum. The process is much more than an arbitrary assignment of values to a matrix in place of careful analysis. Warfare

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<sup>8</sup> Christopher Bellamy, *The Evolution of Modern Land Warfare* (London, 1990), 71.

<sup>9</sup> Ibid., 73.

<sup>10</sup> John Schmitt, *Mastering Tactics: a Tactical Decision Game Workbook* (Quantico: Marine Corps Association, 1994), 83.

obviously is not a game and cannot be conducted without the realization that national existence is dependent upon successful war strategies.

The matrix in figure eight below is the result of the mission analysis applied in appendix two to determine the military worth values of each of the opposing courses of action. There is no software involved or magic solution to determining these values. The ten-step method drives the commander or his staff to prioritize their efforts in systematic manner. Most staffs find it difficult to reach a consensus about the effects achieved by each friendly and enemy encounter. Discussing the rationale behind the analysis is one of the values of the mission analysis process. Not all members of a staff need agree to develop the rank ordering of anticipated effects. The value of the ten-step process is it requires a staff to consider effects as an integral part of the decision-making process, rather than focusing on the plan. Further, the ten-step method breaks down a complicated process into sequential steps that should be manageable for a staff.

Player 1, Germans	Player 2, Russians						Maximin
	COA 1 Attack North	COA 2 Attack South	COA 3 Coordinated Attack	COA 4 Attack North Fix South	COA 5 Attack South Fix North	COA 6 Defend in depth	
COA 1 Attack North, Fix South	24	23	22	3	15	2	2
COA 2 Attack South, Fix North	16	17	11	7	8	1	1
COA 3 Defend in Place	13	12	6	5	4	14	4
COA 4 Defend Along Vistula	21	20	19	10*	9*	18	9*
Minimax	24	23	22	10*	15	18	

Figure 8, Tannenberg payoff matrix

Construction of the matrix permits the decision-maker to consider systematically the merits of the courses of action in relation to the enemy's capabilities. With the matrix constructed, find the optimal solution. Naturally, the first step is to attempt to simplify the problem by checking for dominance. In this example, German course of action four is dominant over German courses of action two and three. Therefore, eliminate these courses of action from further consideration because the Germans obtain better effects against all Russian capabilities if they execute course of action four than if they select either course of action two or three. Historical speculation that the Germans may have done better against the French had they defended along the Vistula River finds support in this mission analysis. From Player Two's perspective, Russian course of action four dominates the first three Russian courses of action. Remembering that the payoff s in the chart reflect losses to Player Two, the payoffs for Russian courses of action one, two and three are higher than Russian course of action four. Therefore, for every possible German course of action, the Russians obtain better results employing course of action number four. The Commander has simplified his decision utilizing game theory. Thus, the original matrix, in figure eight above, reduces to the 2x3-matrix show in figure 9 below by applying the rules of dominance.

		Player 2, Russians			
		COA 4	COA 5	COA 6	
Player 1, Germans	Attack North	Attack North	Attack South	Defend	Maximin
	Fix South	Fix South	Fix North	in depth	
COA 1					
Attack North, Fix South	3	15	2	2	
COA 4					
Defend Along Vistula	10*	9*	18	9*	
Minimax	10*	15	18		

Figure 9, Tannenberg courses of action matrix reduced by dominance.

Game theory supports the commander by providing a model by which to organize the information gained in the estimate of the situation. Figure nine has not made the commander do anything. The chart merely portrays the expected results of enemy and friendly confrontations with values based on the commander's analysis. These values are referred to as payoffs in game theory or military worth when describing effects of military operations. Dr. Von Neumann suggests in his work *Theory of Games and Economic Behavior*, that the relative value of a course of action outcome need not be determined mathematically to facilitate game theory.<sup>11</sup> Organizing the relative value of the outcomes in a relative scale of superior, excellent, good, fair, loss, defeat, etc. can still organize the commander's analysis into a format to determine the best strategy. The assignment of numerical values to the rankings in this example enables the commander to use a computer model to reveal the optimal solution.

To reveal the optimal solution, the commander must perform additional analysis. The commander can apply the Maximin and Minimax theories to the matrix to determine if a saddle point exists and a pure strategy. Examining figure nine reveals, the Maximin is nine, and the Minimax is ten. This indicates that there is no saddle point and the optimal solution must be a mixed strategy. The lack of a saddle point further indicates that this is a closed game because each side could benefit from learning or

deceiving the other about their intentions. Hence, military concepts of deception, secrecy, and intelligence have some value to both players. Like the previous example, this problem can be solved graphically within the limitations of a two-dimensional graph because the matrix has been reduced to a two by  $n$  matrix. Specifically, there are three remaining courses of action for the Russians and two courses of action for the Germans. In the case presented, dominance reduced the German courses of action to two. However, humans do not make decisions solely based on mechanical analysis. In this case, German political pressures made withdrawal to the Vistula extremely undesirable. The effects of the encounters did not consider the political consequences of the encounter but the physical effects on the opposing force. Defining the effects to include political considerations alter the relative value of a course of action. Consequently, it is reasonable to resolve the matrix assuming that it was not reduced. Finding the solution, therefore, requires Algebraic methods and linear programming to determine the optimal solution in this case.

Several computer programs are available to solve two person zero sum games. One program that is currently available, free on the internet to solve simple two person zero sum games, is the General Algebraic Modeling System (GAMS) program.<sup>12</sup> The program requires the planner to establish the matrix and determine the relative values for each combination in the matrix. Having applied the proposed ten-step method, the matrix is complete. The commander or staff can now utilize the computer model to determine the optimal solution. This reduces the mental challenge of resolving the matrix manually and saves time for the commander. By reducing the need to perform the mathematics the GAMS computer program provides an attractive means to reduce resistance to using game theory in the military. Not all military planners are capable of solving linear programming problems under a poncho with a flashlight, nor should they be required to do so. The planner without a math background may be more inclined to try a computer model than endure the mental rigors of solving for the optimal solution graphically or

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<sup>11</sup> COL Oliver Haywood, *Military Doctrine of Decision and the Von Neumann theory of Games*, (Santa Monica: Rand Corporation, 1950), 37.

algebraically. However, GAMS utilization may create resistance from those that are not comfortable with computers. If the army at large adopted game theory as a means of selecting the best strategy, army programmers could construct a program similar to GAMS. In this analysis, GAMS illustrates the ability of computer models to solve zero sum games in military decision-making. The complete GAMS solution printout of this matrix is included in appendix three. The original matrix from figure eight was used to demonstrate the value of a computer program to solve a complex matrix. Again, this matrix is a 4 x 6 matrix. Discussion of the results addresses the concepts of dominance that reduced the matrix to a 2 x 3 matrix in figure nine.

In this case, the computer generated optimal solution for the Germans is a mixed strategy of course of action one and course of action four. The probabilities associated with courses of action one and four are .077 and .923 respectively. These values indicate the Germans should select course of action four, defend along the Vistula River nearly all the time, 92.3 percent of the time. The matrix produces a payoff value of 9.462, which is higher than the Maximin value of nine associated with a pure strategy of course of action four alone. The values indicate the Germans benefit by maintaining secrecy, and employing deception. The Germans could benefit by deceiving the Russians into believing that the Germans intended to attack in the North and fix in the South. Considering the significance of this recommendation, it follows that the Russians would be less effective attacking the Germans along the Vistula River if the Russians believed that they had to defend in the North against a possible attack. The recommended Russian optimal strategy is course of action four, 46.2 percent of the time, and course of action five, 53.8 percent of the time. Referring back to figure nine, this implies that the Russians would do nearly as well attacking in the North or South while fixing the Germans on the other flank. From figure nine, the Russians have an expected loss of ten if course of action four were executed as the pure Minimax solution. By mixing their strategy, the Russian Commander can lower potential loss of 9.462, or do better if they maintain secrecy and deceive the Germans about their intentions. The matrix

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<sup>12</sup> <http://gilbreth.ecn.purdue.edu/~rardin/gams/notes.html> GAMS program.

construction could have indicated positive values for a Russian victory while still achieving the same relative value for each course of action.

The fact that the optimal solution value for the Russians is a negative number does not imply that the Russians would lose and the Germans would win. Since the chart values do not indicate the significance of the mission analysis conducted, commanders need to consider the significance of the values obtained in the matrix. In this case, the Russians would benefit from coordinating their efforts, creating the perception that the two armies were poised to attack either North or South in support of each other. Understanding that the optimal German course of action is to defend along the Vistula and gain advantage of interior lines while fighting the French on another front, it follows that the Russian optimal course of action involves consolidating resources and attacking in the North to defeat the Germans. As previously discussed, Russian numerical superiority could have enabled the Russians to overwhelm the Germans if the Russian armies had united. Therefore, the recommended course of action to fight united is consistent with military logic. Having utilizing the ten-step method to determine the optimal course of action for a historical example, one can now consider some of the broader implications of game theory to military decision-making.

## **VALUE OF GAME THEORY TO MILITARY DECISION-MAKING**

### **Capabilities Based Planning**

In the aforementioned examples, the commander based the courses of actions for each of the players on their capabilities. This is similar to the current doctrine that focuses on enemy capabilities as well as intentions. Since the time of Sun Tzu, military leaders understood the importance of understanding the enemy. To paraphrase a translation of his works, “know the enemy and know yourself and you need not fear the outcome of a hundred battles”.<sup>1</sup> To emphasize the point, the selection of

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<sup>1</sup> Sun Tzu, *The Art of War*, Ralph d. Sawyer translation (Boulder, CO: Westview Press, 1994), 179.

courses of action occurs after the commander's estimate of the situation or mission analysis. The study of doctrinal templates, orders of battle, and intelligence gathered all provide data that improves the commander's situational awareness and decreases uncertainty. Two person zero sum game theory is a tool for the commander to determine the optimal solution in an uncertain situation. With a complex understanding of what the enemy is capable of doing the commander can choose the best strategy to accomplish his mission.

A thorough understanding of the enemy is what challenges two of the key assumptions of the zero sum game. Critics of applying game theory to military situations claim that the opposing forces rarely share the same perspective so the assumptions of game theory are invalid. The first assumption is that the payoff matrix reflects payoffs to Player One and assumes that a gain for one player is a loss to the other. This assumption infers that both players see the situation similarly. Consider the Gulf War fought by the allies against Iraq. The allies claimed that they achieved all campaign objectives and the Iraqi forces had been defeated. From the Iraqi point of view, the Iraqi government of Saddam Hussein was able to remain in power and the allied armies never challenged Iraq for power in the streets of Baghdad. The Iraqi's saw this as a victory because they were able to withstand the allied attacks and remain in power. Clearly, the two opponents do not agree that the gain for one was at the loss of the other. Can there be two winners in modern warfare? This question exemplifies the challenge of understanding the enemy and his motivation to ensure that courses of action obtain their desired effects.

The second assumption of game theory is that of rational opponents. The rational actor model assumes that each opponent in a game is acting in his or her best interest to achieve his or her goals. The motivation of an opponent and the societal values of his or her society will play a clear role in determining what behavior is rational. Consider the variety of civilizations in the world today. Western ideals of liberty, freedom, and equality differ from the beliefs of many of the Middle Eastern

fundamentalist movements.<sup>2</sup> The variety of beliefs and values in the world make it difficult to construct a universal payoff matrix. If the beliefs and values of opponents are different, then the opponents may define gains and losses differently. The rational actor model accepts that there may be different rules on each side.

Consider first the US attacks with precision munitions against Iraqi radar sites in response to challenges to patrols enforcing the no fly zone over Southern Iraq. Citizens of many Middle Eastern countries perceived the use of precision weapons and long-range engagements as acts of cowardice and a demonstration of the United States aversion to war.<sup>3</sup> The withdrawal of soldiers from Somalia immediately following the dragging of an American Ranger through the streets of downtown Somalia, lends credence to the opinion that the United States is unwilling to risk American lives around the globe.<sup>4</sup> Regardless of the merit of Arab interpretations of the United States military response, not all opponents see the world similarly. Based on their society and values, opponents can rationally conclude that the United States has an aversion to war or will not commit ground troops.

To claim that an opponent is not acting rationally because he has a different viewpoint is not valid. Colonel Edward Mann III in his work *Effects Based Methodology for Joint Operations* further describes this behavior as mirror imaging.<sup>5</sup> In short, he states that an adversary cannot project what is normal for him onto his opponent. Colonel Mann tells the story of the Allied air war planners who failed to target the German electrical grid because they believed that the German power system had built in redundancy, similar to the design of United States power systems. In fact, the German system had far less redundancy or excess capacity than anticipated. Non-zero sum game theory addresses the concept of differences in perspectives of payoffs and is beyond the scope of this paper. Simply stated, a payoff

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<sup>2</sup> Samuel Huntington, *the Clash of Civilizations* (New York: Touchstone, 1996), 213.

<sup>3</sup> Ibid., 217.

<sup>4</sup> James Hoge, *How Did This Happen? Terrorism and the New War*, (Public Affairs: New York, 2001) 230.

<sup>5</sup> COL Edward C. Mann III, *Effects Based Methodology for Joint Operations*, Cadre Paper No.15 (Maxwell Air Force Base: Air University Press, 2002), 18.

matrix is constructed for each player, rather than payoffs in terms of Player One. However, focusing on the opposing capabilities and the effects of these combinations supports utilization of two person zero sum game theory. The argument against the validity of two person zero sum game theory may have greater implications at the strategic level than at the tactical level. At the tactical level, a loss to one side has a direct benefit to the other.

Current doctrine directs an estimate of the enemy capabilities as well as enemy intentions.<sup>6</sup> The commander who bases his strategy on enemy capabilities accounts for all possible enemy actions. The set of enemy intentions should be a sub-set of the larger group of enemy capabilities. Therefore, given adequate resources to account for all possible enemy actions, the commander may do better to focus on enemy capability rather than trying to deduce enemy intentions. Selection of a capabilities based strategy is more conservative than a strategy based on intentions, because the potential gains from anticipating an enemy course of action are greater than the Maximin solution. The problem facing military planners is that seldom are sufficient forces available to cover all possible enemy courses of action and still mass overwhelming force against the enemy. Colonel Haywood uses the example of Pearl Harbor as a case where the United States strategy failed to account for Japanese capability to drop bombs in Hawaii. Had the US strategy been based on the capabilities of the Japanese, defensive measures could have been improved to reduce the threat to the Pacific Fleet.<sup>7</sup> By using the concepts of the two person zero sum game, one can ensure that all the enemy capabilities are addressed and that the pessimistic or worst case scenario has been accounted for. Intelligence that is collected can assist a commander in creating the conditions to capitalize on opponents faults rather than react to his genius. In other words, the use of game theory ensures that the best of the worst-case scenario has been selected. If the enemy attempts to

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<sup>6</sup> United States Army, Field Manual Number 101-5 (Washington, D.C., Government Printing Office, 1997), 5-7.

<sup>7</sup> O.G. Haywood, "Military Decisions and Game Theory", *Journal of the Operations Research Society of America* 4 (November 1954), 377.

do his worst to a commander, then the commander has already planned for this. A commander can improve his results if the enemy selects a less effective course of action.

The School of Advanced Military Studies Arab-Israeli War of 1973 Practicum, Academic Year 2002-2003, provides another example of the utility of game theory to military decision-making. It is important to accept the fact that this was a simulation exercise. The exercise did not attempt to reproduce the history of the war. A brief historical synopsis establishes the scenario sufficiently to discuss the utility of game theory to military decision-making. The Egyptian forces in the South, Jordanians in the East and Syrians in the North surrounded Israel. The Arab League Commander developed a plan to attack first, maximize surprise, and attempt to force Israel back to the pre-1967 Israeli borders. Historically, the Israeli Air Force and Israeli Defense Force mobilization system were influential in facilitating a counter-attack to defeat the Arab League. In the mission analysis, attention centered on these two Israeli capabilities and looked for a strategy to take advantage of surprise to disrupt the Israeli Air Force and mobilization system. The initial effects in the practicum were so significant that the Arab League achieved all their operational objectives by the end of the first turn. Arab League plans eliminated the Israeli capability to achieve air superiority and mobilize. Based on an understanding of enemy capabilities, no Arab league maneuver occurred beyond the coverage of anti-air assets until the Arab League destroyed the Israeli Air Force. Because The Arab league selected a course of action based on worst-case enemy capabilities, even if the Israeli Air Force or mobilization systems survived, the course of action selected would still have been effective. Had the Israeli Defenses been more resilient to the initial attacks, the plan would still have attained the initial operational objectives because his plan accounted for the enemy capabilities, rather than intentions. From this point, the commander was able to benefit from the opportunities created by decreasing the enemy capabilities. By focusing on the enemy's capabilities, his intentions became of lesser importance.

Remember that Maximin theory assumes the worst case based on enemy capabilities in conditions of uncertainty. If one assumes the worst, there should be few bad surprises. This of course

does not eliminate the fog of war or the fact that even the simple is difficult in battle. It does, however, provide commanders with a tool to assist them in military decision-making when selecting a course of action. This approach may seem pessimistic or cautious as opposed to bold and audacious action determined to achieve decisive victory. However, mitigating risks by planning against the worse case scenario ensures that one should do as well or better than planned. This may not achieve decisive victory but it should make it difficult to lose. Best-case plans run aground quickly when the enemy acts unexpectedly. Commanders can develop success sequels and branches to exploit opportunities as the enemy situation becomes clearer. One can suggest that there may be a morale benefit to executing a success sequel over executing a failure sequel. Much of this discussion has focused on the enemy capabilities. Course of action selection based on Maximin theory may be preferred to accepting defeat and reacting with defeat sequels.

## Importance of the Estimate of the Situation

Earlier in the Tannenberg example, the concept of effects determined the military worth values in the payoff matrix. The commander analyzed the combination of each friendly and enemy course of action to determine the effect achieved. These effects were rank ordered from best case to worst-case outcomes. The discussion of effects based operations or EBO has received increased consideration in professional journals. There are several doctrinal definitions of what effects based operations means. The Air Force, Army, Marine Corps, and Joint communities all have differing concepts of what effects based operations means. Understanding the doctrine hierarchy, the Joint definition should be the binding definition. Lieutenant Colonel Allen Batschelet from the J9, Joint Forces Command proposes the following definition of effects based operations: “A process for obtaining a desired strategic outcome or effect on the enemy through the synergistic and cumulative application of the full range of military and nonmilitary capabilities”.<sup>8</sup> Similarly, Dr. Joseph Strange has suggested a new model of linking centers of

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<sup>8</sup> Allan Batschelet, Effects-Based Operations: a New Operational Model, J9 Joint Forces Command.

gravity, critical capabilities, critical requirements, and critical vulnerabilities to mission analysis and decision-making in his work, “Perspectives on Warfighting.”<sup>9</sup> These works reinforce the concept that it is critical to understand the enemy capabilities and understand the effects of friendly actions.

Game theory provides a means to quantify and organize the results of the analysis that determines the rank ordering of the effects of each course of action in relation to the enemy. Too often commanders develop plans based on the greatest potential gain with minimal attention placed on the enemy actions to disrupt these plans. Some argue that the recent discussion surrounding the conduct of the Joint Exercise Millennium Challenge occurred because the planners failed to consider the enemy capabilities and the results of enemy actions applied to the friendly course of action.<sup>10</sup> In effect, the enemy forces were able to destroy nearly the entire fleet using small ships in the early stages of the exercise. The exercise had to be restarted due to the tremendous impact this had on the friendly forces. Again, conceding that simulations are limited and may not truly represent the planning effort. Planning for all possible enemy and friendly actions may seem like a daunting task. The payoff matrix provides a means of organizing the results of a planner’s analysis. The concepts of game theory support selecting the optimal course of action based on an analysis of enemy capabilities. Due to a reluctance to utilize math, some may claim that relying on math to make a military decisions is irresponsible. However, this is not the intended function of the payoff matrix. The better the estimate of the situation and understanding of the enemy obtained by the staff, the better the analysis. The better the analysis, the better the payoff matrix, and subsequent conclusions about courses of action made by the commander. Again, Game theory does not suggest that there is no need for mission analysis; on the contrary, it reinforces the importance of this part of the decision-making process.

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<sup>9</sup> Dr. Joe Strange, (Centers of Gravity and Critical Vulnerabilities, Perspectives on Warfighting, number four, second edition, Marine Corps University Foundation, Quantico, Virginia, 1996),vi.

<sup>10</sup> Mackubin Tomas Owens, Let’s Not Rig Our War Games, *National Review* (29 August 2002).

Another shortfall in game theory comes from the assumption that a mixed strategy will obtain a greater return over time than the Maximin solution. The Maximin solution does not recommend the best possible outcome if careful planning, chance, or luck provide all the right opportunities for success. Additionally, in game theory the results are determined for each game as if the start of each game or turn was identical to the last. In other words, players repeat the same scenario. Over the long run, the results of the optimal solution will be optimal. Rarely in a military situation will the situation exist that can be replicated after a battle. Battles lead to operations and campaigns that are separate independent events when considered in isolation, but are affected by a variety of variables. Rarely will the mission, enemy, terrain, weather, troops, or morale be the same for any two battles. This is a valid difference between game theory and military operations.

Commanders, however, can still use game theory to assist him in organizing and evaluating his options. Consider the example of military events that are repeatable, such as convoy operations or aerial flights. COL Haywood suggests that the game theory may be more applicable at the small unit level.<sup>11</sup> This has interesting potential because, as the numbers of decisions are increased, the benefits of a conservative strategy become more apparent in the end. Further, a conservative strategy does not require a military genius to make it succeed. In effect, it ensures that with proper execution of the plan the commander will not be defeated. Of course, this assumes that the effects of the clash of capabilities have a success option for the friendly player. This assumption is can be made as military leaders generally ensure that they maintain a balance between mission, structure, and resources to prevent failure. As with all mixed strategies, the concepts of deception, espionage and secrecy have utility. They are not new concepts proposed to the military by game theorists. Remembering that military decisions are decisions and not games, game theory is still a valuable tool for military decision-making.

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<sup>11</sup> O.G. Haywood, "Military Decisions and Game Theory," *Journal of the Operations Research Society of America* 4 (November 1954), 382.

Another fact revealed when a payoff matrix has a mixed solution is the importance of intelligence. The emphasis, thus far, focused on a conservative strategy based on capabilities rather than intentions. The lack of a saddle point demonstrates the value of intelligence and discerning the enemy actions or intentions. A mixed solution tells a commander how much of his resources he should allocate to acquiring information based on the potential gain from the mixed solution. In the Tannenberg example, the friendly payoff determined for the Germans defending along the Vistula River was nine when compared to the Russian course of action of attacking in the South and fixing in the North. A payoff value of eighteen could have been achieved if the Russians decided to defend in depth against a German defense along the Vistula. The difference between the two payoff values establishes the relative worth of intelligence to the commander and facilitates an estimate of the resources to allocate to intelligence gathering. The game theory solution demonstrates that a strategy based on enemy intentions can have better results than one based on capabilities. A key point associated with focusing on enemy intentions is that a commander should only adjust his plan after mitigating all risks based on enemy capabilities. A plan based on intentions cannot guarantee success, and may be reckless. Once, a commander has considered the implications of the enemy worst-case scenario and mitigated the risks associated with it, intelligence gathering can eliminate some uncertainty. Clearly stated, if the commander knows the enemy actions, he may be able to improve his effects upon the enemy. Similarly, an analysis of the combinations of capabilities effects may reveal important seams or vulnerabilities that require attention to prevent enemy exploitation.

## **CONCLUSION**

The military decision-making process relies heavily on the mission analysis and a thorough understanding of enemy and friendly capabilities. Game theory supports the importance of this process and provides a valuable tool for the commander to organize his analysis and prioritize the relative value,

or military worth, of these outcomes. Contrary to what the name implies, game theory does not suggest that things should be left to chance or some roll of the dice to determine the optimal strategy. Game theory relies on quantifiable analysis to determine the optimal course of action against a skilled opponent. In fact, a critical assumption of game theory is that the opponent will choose the best course of action to minimize the enemy commander's gains. Game theory is not about a game, it is about decision-making.

Incident to a military decision is a planning decision to build a plan based on an assessment of enemy capabilities or intentions. The bold and aggressive commander develops an optimistic strategy that plans to capitalize on any enemy mistakes by understanding the enemy intentions from the onset. This optimistic strategy in turn assumes more risk if the commander misinterprets his opponent's intentions. In contrast, the commander who bases his strategy on enemy capabilities has the benefit of identifying the worst-case scenario and ensuring that the outcome of events will be no worse than anticipated. Any irrational, unwise, or poorly executed actions by an opponent will present an opportunity for that commander to exploit.

The concept of mixed strategies in game theory has further reinforced the importance of thorough mission analysis and the validity of the military decision-making process. Intelligence, deception, and secrecy all have roots that date back to the start of military operations and are integral parts of game theory. They support the value of a commander varying his actions to prevent the enemy from predicting his chosen course of action. Players benefit by deception in military situations just as they can in a game of poker. Deception aims to mislead the opponent. Deceiving an enemy about the friendly intentions has obvious benefits to the military planner. Similarly, intelligence gained can contribute to a commander's decision-making process by reducing uncertainty. Reliable intelligence facilitates modifying a strategy based on enemy capabilities when the opponent's actions become clear. Capabilities based planning ensures the outcome will be no worse than the Maximin solution. The same is not true for a solution based solely on enemy intentions. Any error in the analysis of enemy intentions can dramatically alter the outcome of events. The capabilities based analysis provides the commander a much firmer foundation to

build on as the situation develops. Thus, the concepts of game theory can assist the military decision maker to make better decisions.

The proposed ten-step process to determine the military worth for a two person zero sum game payoff matrix permits the incorporation of the concepts of game theory without delving into the mathematical realm of proofs and theorems. The intent is to translate the abstract concepts of game theory to a well-defined process for organizing information to enhance military decision-making. This study offers a model for the military commander to augment the military decision-making process. The model organizes the results of mission analysis focused on the effects of the clash of opposing courses of action. As uncertainty increases in military operations, game theory provides a valuable means to simplify complex problems facing the military commander. The military, therefore, should adopt game theory as an organizational means to enhance the commander's ability to organize information and make better decisions under uncertainty.

This study has established a basic understanding of two person zero sum games and developed several practical examples to demonstrate the utility of game theory to military decision-making. The use of the Tannenberg scenario has demonstrated the concepts of mixed strategies and the importance of capabilities based planning. The ten-step process has shown the ability to use game theory to determine an optimal solution and organize information that has been obtained through the military decision-making process. The commander gains insight about the relative importance of secrecy, espionage, deception, and intelligence based on the optimal solution. Perhaps, the operations research community will develop these ideas beyond the scope of this study and leverage technology to enhance the military decision-making process. Too often, mission analysis is completed before a complete understanding of the enemy or situation can be obtained. Time is still as unforgiving a resource as it was in the days of Napoleonic warfare and time governs much of what is possible. Many systems have emerged that improve a commander's situational awareness. Some claim that these sensors provide an overflow of information that takes more of a commander's time and hinders his ability to make a decision. Perhaps these systems

will assist the commander to eliminate uncertainty. Until that day, game theory offers a means for the military commander to organize the results of his analysis and make better decisions. As Colonel Haywood stated fifty-three years ago, “If a commander is not prepared to make a matrix of opposing strategies for the situation, he isn’t prepared to make a decision.”<sup>1</sup>

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<sup>1</sup>O.G. Haywood, “Military Decisions and Game Theory,” *Journal of the Operations Research Society of America* 4 (November 1954), 384.

## APPENDIX ONE

### STEPS FOR SOLVING TWO PERSON ZERO SUM GAMES

These steps present a method to solve problems using two person zero sum game theory. In this study, this methodology determined the optimal solution for each player. The following steps serve as a brief reminder of this methodology to find the optimal solution for each player for a two person zero sum game matrix.

- 1) Identify the two players. Player One will represent the Blue Player and Player Two will represent the Red Player. Coalitions can be formed to represent one player but they must as a team act as one player.
- 2) Identify the number of possible courses of action for each player. The Blue Player, Player One, is the row player and his courses of action become the rows of the matrix. The Red Player, Player Two, is the column player. The Red Player's courses of action become the columns. A course of action sketch can be helpful to maintain clarity of each course of action.
- 3) Construct a matrix listing the Blue Player's courses of action on the rows and the Red Player's courses of action on the columns.
- 4) Place the payoff values in the matrix for each combination of opposing course of action for each player. All values should be in terms of payoffs to the Blue Player. If these values are not provided, based on the mission analysis utilize the ten-step process for matrix development presented in appendix two.
- 5) Apply the theory of dominance to remove inferior courses of action from further consideration and simplify the matrix.
- 6) Find the Maximin for the row player and the Minimax for the column player. Check this solution for a saddle point. If a saddle point exists then the Maximin and Minimax indicates a pure solution for the optimal solution of the game. Each player should select the course of action

indicated by the Maximin and Minimax. If no saddle point exists then a mixed strategy will need to be found for each player.

- 7) Find the optimal mixed strategy solution. If the matrix has been reduced to two courses of action for either player, the graphical solution can be used. If more than two courses of action remain for both players or if preferred, an algebraic solution can be found utilizing the GAMS software. An example solution from the GAMS program is located in appendix three along with the procedures for using the games program.
- 8) Refer to the graphical solution or the GAMS solve summary to determine the optimal solution for each player. Interpret this data to determine the optimal mixed strategy for each player.

## **APPENDIX TWO**

### **THE TEN-STEP PROCESS FOR DETERMINING THE MILITARY WORTH FOR A TWO PERSON ZERO SUM GAME PAYOFF MATRIX**

The following slides represent a PowerPoint presentation to aid in understanding the application of the proposed ten-step method for determining the military worth values for a two person zero sum game. They have been adapted to JPEG format for inclusion in this appendix. The challenge for the decision maker is to incorporate the information obtained in mission analysis consistently to clarify the problem. The following ten steps are presented in support of the mission analysis process and attempt to aid course of action development, comparison, and selection. This method utilizes the two person zero sum game to focus on the effects generated by the opposing courses of action. It is a capabilities based analysis dependant upon the military decision maker's understanding of the scenario. The optimal solution for most scenarios can be determined once the values of military worth have been determined for the matrix. Several other methods exist to determine military worth. This ten-step process is presented to improve military decision-making and encourage other imaginative solutions to complex problems.

## Steps to determine military worth values for a two person zero sum game payoff matrix

- 1) Select the best-case friendly course of action to achieve a decisive victory.
- 2) Rank order the friendly courses of action from the best effects possible, to the worst effects possible.
- 3) Rank order the effects of enemy courses of action from the best to the worst across each row, in terms of the friendly player.
- 4) Determine whether the effects of the enemy course of action will result in a potential loss, tie, or win for the friendly player for every combination across each row.
- 5) Place the product of the number of rows multiplied by the number of columns in the box corresponding to the best case scenario for player one.

*Figure 1, first five steps to determine military worth values for a two person Zero Sum Game payoff matrix*

## Steps to determine military worth values for a two person zero sum game payoff matrix (Continued)

- 6) Rank order all of the combinations from best case to worst case that are marked win, from step 4. Use the product obtained in step five as the highest number and assign values in descending order based on the best to the worst case effects anticipated for each combination marked win.
- 7) Place the number one in the box corresponding to the worst case effects anticipated in terms of player one.
- 8) Rank order all combinations marked loss, from step 4, from the worst case to the best case.
- 9) Rank order all remaining even combinations from the best to the worst case.
- 10) Transcribe the matrix into a conventional format with each course of action listed in ascending order while maintaining the appropriate values for each combination

*Figure 2, Steps five to ten to determine military worth values for a two person Zero Sum Game payoff matrix*

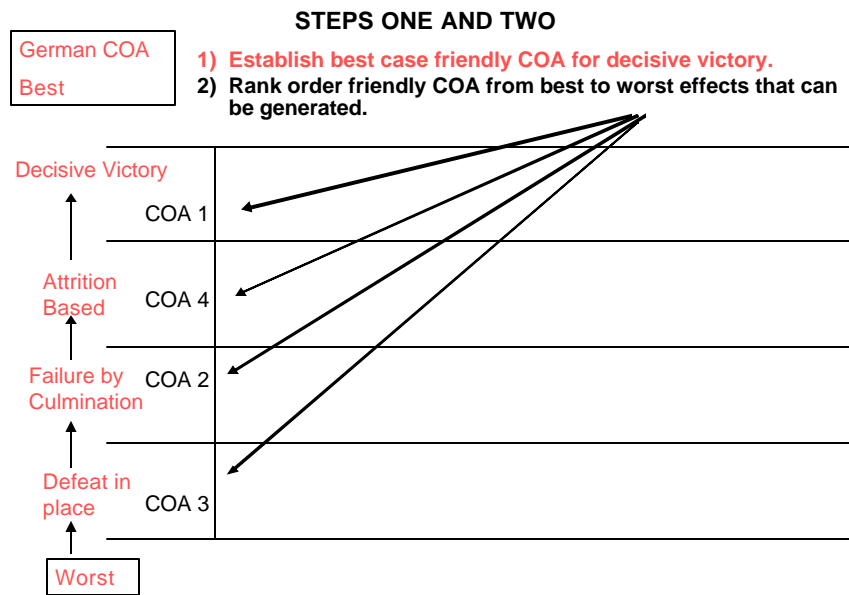


Figure 3, Steps one and two to determine military worth values for a two person Zero Sum Game payoff matrix

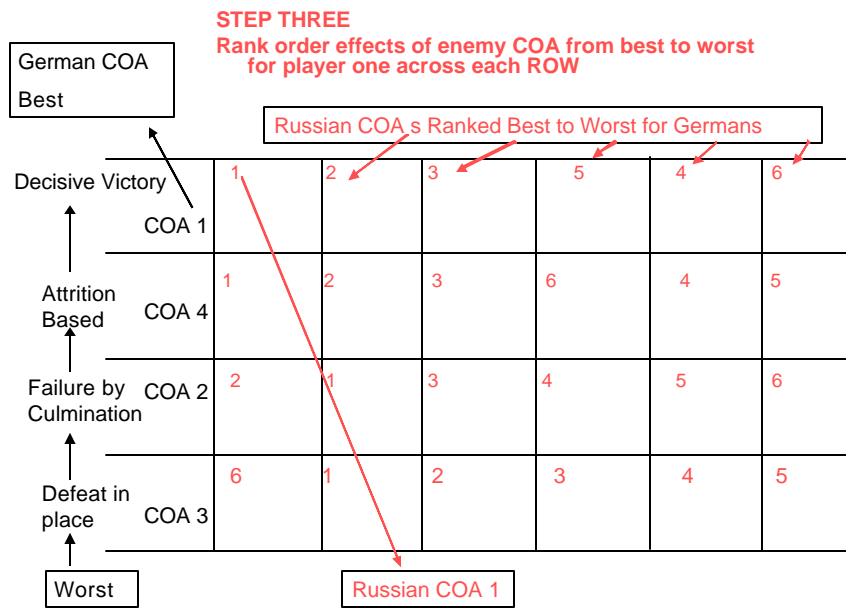


Figure 4, Step three to determine military worth values for a two person Zero Sum Game payoff matrix

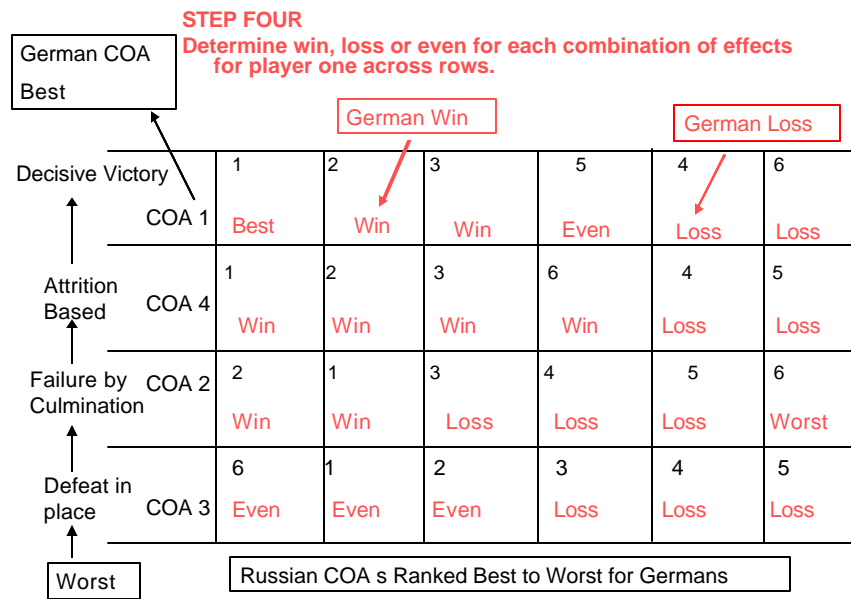


Figure 5, Step four to determine military worth values for a two person Zero Sum Game payoff matrix

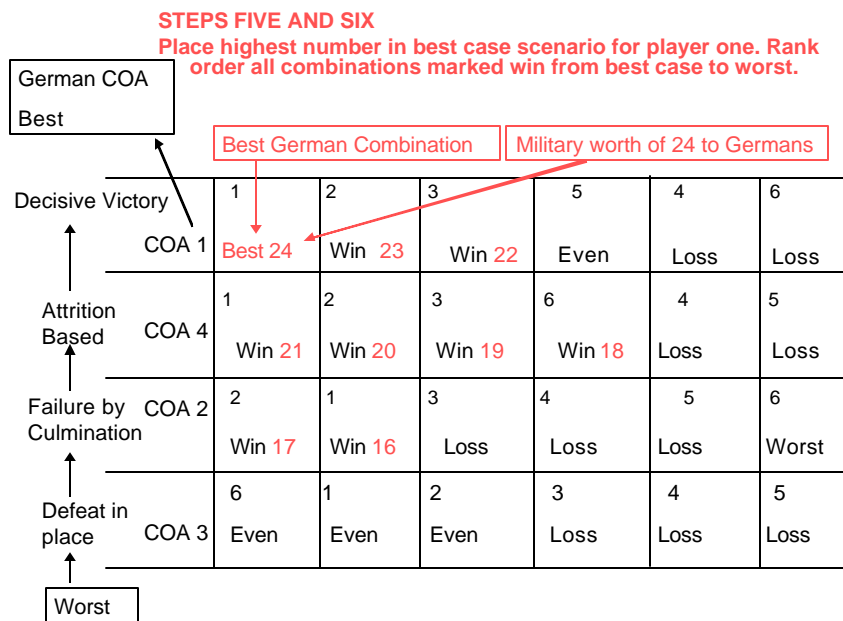


Figure 6, Steps five and six to determine military worth values for a two person Zero Sum Game payoff matrix

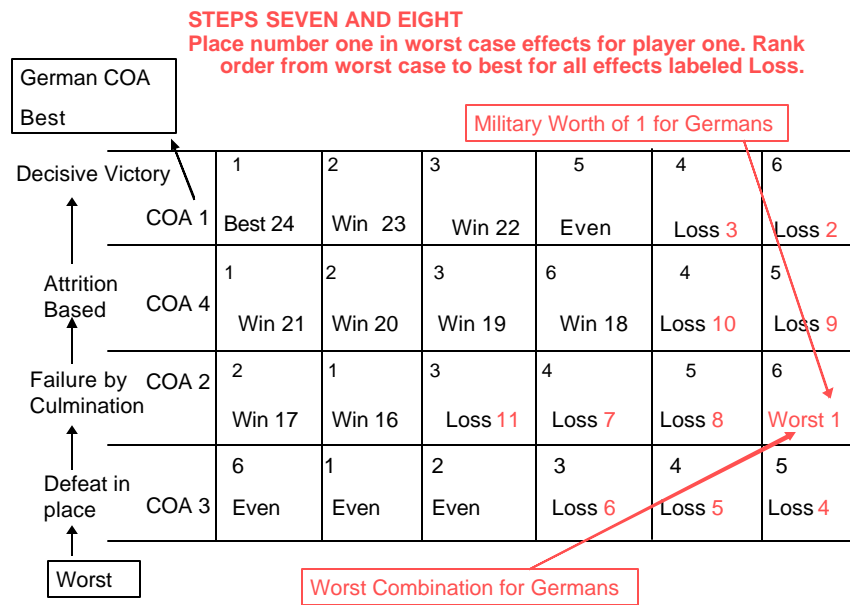


Figure 7, Steps seven and eight to determine military worth values for a two person Zero Sum Game payoff matrix

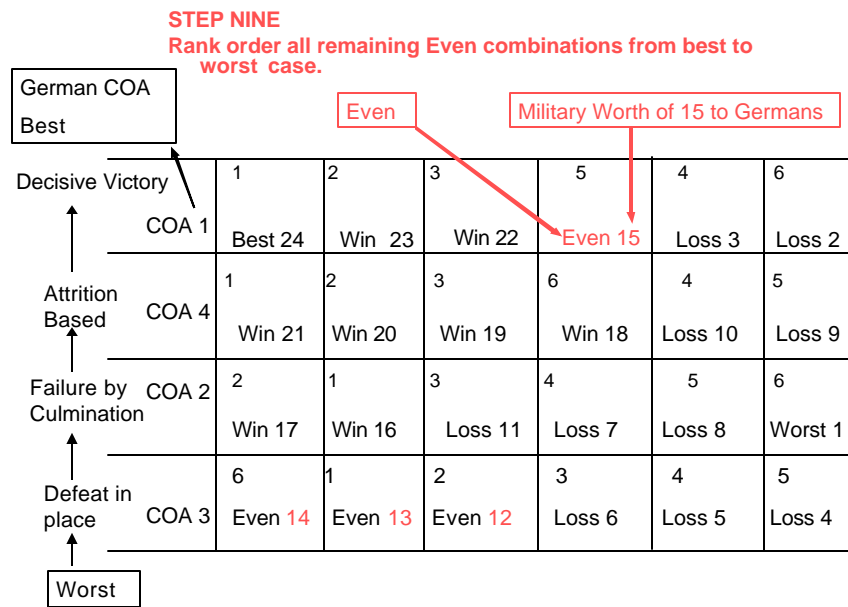


Figure 8, Step nine to determine military worth values for a two person Zero Sum Game payoff matrix



Figure 9, completed matrix to determine military worth values for a two-person Zero Sum Game payoff matrix

### STEP TEN

Transcribe matrix into conventional format (COA1,2,3...) maintaining appropriate values for each combination.

		Player 2, Russians					
Player 1, Germans		COA 1	COA 2	COA 3	COA 4	COA 5	COA 6
		Attack North	Attack South	Coordinated	Attack North	Attack South	Defend
				Attack	Fix South	Fix North	in depth
COA 1							
Attack North, Fix South	24	23	22	3	15	2	2
COA 2							
Attack South, Fix North	16	17	11	7	8	1	1
COA 3							
Defend in Place	13	12	6	5	4	14	4
COA 4							
Defend Along Vistula	21	20	19	10*	9*	18	9*
Minimax	24	23	22	10*	15	18	

Figure 10, Step ten completed matrix transcribed into conventional format.

## **APPENDIX THREE**

### **GAMS PROGRAM SOLUTION FOR THE TANNENBURG EXAMPLE**

The first two pages of this printout represent the data from the Tannenberg example payoff matrix. To utilize GAMS to get to this point the GAMS software must be installed and opened on the computer. Once opened a game file must be opened from the file menu window available in the upper left corner of the program window. For this problem, the file vngame was opened. Dr. David Bitters, Professor, US Army Command and General Staff College, wrote the file vngame program. The number of courses of action for Player One is indicated in line six by X1, X2, X3, and X4. This represents courses of action one through four for Player One. Line seven indicates the courses of action for Player Two and they are identified by Y1, Y2, Y3, Y4, Y5, and Y6. The matrix developed in lines 12 through 16 represent the values of military worth developed in appendix two for Player One based on the mission analysis. The values in lines 22 through lines 26 represent the payoff values in terms of Player Two. They are the values of the matrix developed in appendix two multiplied by negative one. This is due to the concept of a zero sum game that a gain to one player is a loss to the other. With this completed, save the file under a new name using the pull down file menu option. From the file menu options now, select file run. This will take a few seconds. A summary message will appear on the screen in blue similar to the message at the top of page three below. Double click on the solve summary message and print out similar to the following pages will appear on your computer screen.

The solve summary portion listed on page eight of the printout lists the value of the game as variable V1 and indicates 9.462 as the value that Player One could expect from the game based on the optimal solution found. The probabilities for player 1 COA are listed on page eight and represent the percent of time that Player One should play strategies X1-X4. In this case, strategy X1 corresponds to COA 1 and should be selected 7.7 percent of the time. Further, X4 corresponds to COA 4 and should be

selected 92.3 percent of the time. For Player Two the probabilities are shown as .462 for Y4 and .538 for Y5. This indicates that the optimal mixed strategy for Player Two is to play COA4 46.2 percent of the time and COA 5 53.8 percent of the time.

## A QUADRATIC PROGRAMMING MODEL FOR EQUILIBRIUM POINTS OF NONZERO SUM GAMES

```

3
4   SETS
5
6       I      player 1 COA / X1, X2, X3, X4/
7       J      player 2 COA / Y1, Y2, Y3, Y4, Y5, Y6/;
8
9
10      TABLE      A(I,J)  player 1 payoff matrix
11
12                  Y1      Y2      Y3      Y4      Y5      Y6
13      X1      24      23      22      3      15      2
14      X2      16      17      11      7      8      1
15      X3      13      12      6      5      4      14
16      X4      21      20      19      10      9      18;
17
18
19      TABLE      B(I,J)      player 2 payoff matrix
20
21
22                  Y1      Y2      Y3      Y4      Y5      Y6
23      X1      -24      -23      -22      -3      -15      -2
24      X2      -16      -17      -11      -7      -8      -1
25      X3      -13      -12      -6      -5      -4      -14
26      X4      -21      -20      -19      -10      -9      -18;
27
28
29      VARIABLES
30
31          V1      game value for player 1
32          W1      game value for player 2
33          OBJV    value of objective function
34          P(I)    probabilities for player 1 COA
35          Q(J)    probabilities for player 2 COA;
36
37      POSITIVE VARIABLES
38
39          P
40          Q;
41
42
43      EQUATIONS
44
45          PSUM      p-values must add to one
46          QSUM      q-values must add to one
47          OBJF      objective function
48          XCON(I)   player 1 equilibrium constraints
49          YCON(J)   player 2 equilibrium constraints;
50
51      PSUM..      SUM(I,P(I)) =E= 1.0;

```

```

52  QSUM..      SUM(J,Q(J)) =E= 1.0;
53  XCON(I)..   SUM(J,B(I,J)*Q(J))-W1 =G= 0;
54  YCON(J)..   SUM(I,P(I)*A(I,J))-V1 =G= 0;
55  OBJF..      V1+W1 =E= OBJV;
56
57  MODEL       VNGAME /ALL/;
GAMS 2.50E      Windows NT/95/98                      02/12/03 10:40:42  PAGE
2
A QUADRATIC PROGRAMMING MODEL FOR EQUILIBRIUM POINTS OF NONZERO SUM GAMES

58  SOLVE       VNGAME USING LP MAXIMIZING OBJV;

```

COMPILATION TIME = 0.078 SECONDS 0.7 Mb WIN-19-115  
 GAMS 2.50E Windows NT/95/98 02/12/03 10:40:42 PAGE 3  
 A QUADRATIC PROGRAMMING MODEL FOR EQUILIBRIUM POINTS OF NONZERO SUM GAMES  
 Equation Listing SOLVE VNGAME USING LP FROM LINE 58

---- PSUM =E= p-values must add to one

PSUM..  $P(X1) + P(X2) + P(X3) + P(X4) =E= 1$  ; (LHS = 0, INFES = 1 \*\*\*)

---- QSUM =E= q-values must add to one

QSUM..  $Q(Y1) + Q(Y2) + Q(Y3) + Q(Y4) + Q(Y5) + Q(Y6) =E= 1$  ;

(LHS = 0, INFES = 1 \*\*\*)

---- OBJF =E= objective function

OBJF..  $V1 + W1 - OBJV =E= 0$  ; (LHS = 0)

---- XCON =G= player 1 equilibrium constraints

XCON(X1)..  $- W1 - 24*Q(Y1) - 23*Q(Y2) - 22*Q(Y3) - 3*Q(Y4) - 15*Q(Y5)$

$- 2*Q(Y6) =G= 0$  ; (LHS = 0)

XCON(X2)..  $- W1 - 16*Q(Y1) - 17*Q(Y2) - 11*Q(Y3) - 7*Q(Y4) - 8*Q(Y5) - Q(Y6)$

$=G= 0$  ; (LHS = 0)

XCON(X3)..  $- W1 - 13*Q(Y1) - 12*Q(Y2) - 6*Q(Y3) - 5*Q(Y4) - 4*Q(Y5)$

$- 14*Q(Y6) =G= 0$  ; (LHS = 0)

REMAINING ENTRY SKIPPED

---- YCON =G= player 2 equilibrium constraints

YCON(Y1)..  $- V1 + 24*P(X1) + 16*P(X2) + 13*P(X3) + 21*P(X4) =G= 0$  ; (LHS = 0)

YCON(Y2)..  $- V1 + 23*P(X1) + 17*P(X2) + 12*P(X3) + 20*P(X4) =G= 0$  ; (LHS = 0)

YCON(Y3)..  $- V1 + 22*P(X1) + 11*P(X2) + 6*P(X3) + 19*P(X4) =G= 0$  ; (LHS = 0)

---- V1 game value for player 1

V1  
(.LO, .L, .UP = -INF, 0, +INF)  
1 OBJF  
-1 YCON(Y1)  
-1 YCON(Y2)  
-1 YCON(Y3)  
-1 YCON(Y4)  
-1 YCON(Y5)  
-1 YCON(Y6)

---- W1 game value for player 2

W1  
(.LO, .L, .UP = -INF, 0, +INF)  
1 OBJF  
-1 XCON(X1)  
-1 XCON(X2)  
-1 XCON(X3)  
-1 XCON(X4)

---- OBJV value of objective function

OBJV  
(.LO, .L, .UP = -INF, 0, +INF)  
-1 OBJF

---- P probabilities for player 1 COA

P(X1)  
(.LO, .L, .UP = 0, 0, +INF)  
1 PSUM  
24 YCON(Y1)  
23 YCON(Y2)  
22 YCON(Y3)  
3 YCON(Y4)  
15 YCON(Y5)  
2 YCON(Y6)

P(X2)  
(.LO, .L, .UP = 0, 0, +INF)  
1 PSUM  
16 YCON(Y1)  
17 YCON(Y2)  
11 YCON(Y3)  
7 YCON(Y4)  
8 YCON(Y5)

P probabilities for player 1 COA

```

1      YCON(Y6)

P(X3)      (.LO, .L, .UP = 0, 0, +INF)
1      PSUM
13     YCON(Y1)
12     YCON(Y2)
6      YCON(Y3)
5      YCON(Y4)
4      YCON(Y5)
14     YCON(Y6)

```

REMAINING ENTRY SKIPPED

---- Q probabilities for player 2 COA

```

Q(Y1)      (.LO, .L, .UP = 0, 0, +INF)
1      QSUM
-24     XCON(X1)
-16     XCON(X2)
-13     XCON(X3)
-21     XCON(X4)

```

```

Q(Y2)      (.LO, .L, .UP = 0, 0, +INF)
1      QSUM
-23     XCON(X1)
-17     XCON(X2)
-12     XCON(X3)
-20     XCON(X4)

```

```

Q(Y3)      (.LO, .L, .UP = 0, 0, +INF)
1      QSUM
-22     XCON(X1)
-11     XCON(X2)
-6      XCON(X3)
-19     XCON(X4)

```

REMAINING 3 ENTRIES SKIPPED

MODEL STATISTICS

BLOCKS OF EQUATIONS	5	SINGLE EQUATIONS	13
BLOCKS OF VARIABLES	5	SINGLE VARIABLES	13
NON ZERO ELEMENTS	71		

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EXECUTION TIME = 0.109 SECONDS 1.4 Mb WIN-19-115  
GAMS 2.50E Windows NT/95/98 02/12/03 10:40:42 PAGE 7  
A QUADRATIC PROGRAMMING MODEL FOR EQUILIBRIUM POINTS OF NONZERO SUM GAMES

S O L V E S U M M A R Y

MODEL	VNGAME	OBJECTIVE	OBJV
TYPE	LP	DIRECTION	MAXIMIZE
SOLVER	BDMLP	FROM LINE	58

\*\*\*\* SOLVER STATUS 1 NORMAL COMPLETION  
\*\*\*\* MODEL STATUS 1 OPTIMAL  
\*\*\*\* OBJECTIVE VALUE 0.0000

RESOURCE USAGE, LIMIT	0.031	1000.000
ITERATION COUNT, LIMIT	6	10000

BDMLP 1.2 May 18, 2000 WIN.BD.NA 19.3 055.039.038.WAT

(A. Brooke, A. Drud, and A. Meeraus,  
Analytic Support Unit,  
Development Research Department,  
World Bank,  
Washington, D.C. 20433, U.S.A.)

Work space allocated -- 0.04 Mb

EXIT -- OPTIMAL SOLUTION FOUND.

	LOWER	LEVEL	UPPER	MARGINAL
---- EQU PSUM	1.000	1.000	1.000	9.462
---- EQU QSUM	1.000	1.000	1.000	-9.462
---- EQU OBJF	.	.	.	-1.000

PSUM p-values must add to one  
QSUM q-values must add to one  
OBJF objective function

---- EQU XCON player 1 equilibrium constraints

	LOWER	LEVEL	UPPER	MARGINAL
X1	.	.	+INF	-0.077
X2	.	1.923	+INF	.
X3	.	5.000	+INF	.
X4	.	.	+INF	-0.923

---- EQU YCON player 2 equilibrium constraints

	LOWER	LEVEL	UPPER	MARGINAL
Y1	.	11.769	+INF	.
Y2	.	10.769	+INF	.
Y3	.	9.769	+INF	.
Y4	.	.	+INF	-0.462
Y5	.	.	+INF	-0.538

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A QUADRATIC PROGRAMMING MODEL FOR EQUILIBRIUM POINTS OF NONZERO SUM GAMES

EQU YCON player 2 equilibrium constraints

	LOWER	LEVEL	UPPER	MARGINAL
Y6	.	7.308	+INF	.

	LOWER	LEVEL	UPPER	MARGINAL	
----	VAR V1	-INF	9.462	+INF	.
----	VAR W1	-INF	-9.462	+INF	.
----	VAR OBJV	-INF	.	+INF	.

V1 game value for player 1  
W1 game value for player 2  
OBJV value of objective function

---- VAR P probabilities for player 1 COA

	LOWER	LEVEL	UPPER	MARGINAL
X1	.	0.077	+INF	.
X2	.	.	+INF	-1.923
X3	.	.	+INF	-5.000
X4	.	0.923	+INF	.

---- VAR Q probabilities for player 2 COA

	LOWER	LEVEL	UPPER	MARGINAL
--	-------	-------	-------	----------

Y1	.	.	+INF	-11.769
Y2	.	.	+INF	-10.769
Y3	.	.	+INF	-9.769
Y4	.	0.462	+INF	.
Y5	.	0.538	+INF	.
Y6	.	.	+INF	-7.308

\*\*\*\* REPORT SUMMARY :

0	NONOPT
0	INFEASIBLE
0	UNBOUNDED

EXECUTION TIME = 0.000 SECONDS 0.7 Mb WIN-19-115

USER: GAMS Development Corporation, Washington, DC G871201:0000XX-XXX  
Free Demo, 202-342-0180, sales@gams.com, www.gams.com DC9999

\*\*\*\* FILE SUMMARY

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LST

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